Computer Vision CITS4240

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Convolution and Correlation

Convolution and correlation are the methods by which we apply spatial filters to an image.

Convolution

Convolution of two functions $f(x)$ and $g(x)$, usually denoted by $f(x) * g(x)$ or $f(x) \otimes g(x)$, is given by

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(\alpha) g(x - \alpha) d\alpha$$

where, in our context, $f(x)$ is typically the image, $g(x)$ is the filter, and α is a dummy variable of integration.

In the discrete case the equation above becomes a summation

$$f(x) * g(x) = \sum f(\alpha) g(x - \alpha).$$

Informally: convolution involves 'flipping' the second function about the origin and then sliding it past the first function, integrating the product of the overlapping sections of the functions.

Figures 1 and 2 give graphical illustrations of the convolution process for the continuous and discrete cases.

Correlation

Correlation of two functions $f(x)$ and $g(x)$, usually denoted by $f(x) \circ g(x)$, is given by

$$f(x) \circ g(x) = \int_{-\infty}^{\infty} f^*(\alpha) g(x + \alpha) d\alpha$$

where $f^*(x)$ denotes the complex conjugate of $f(x)$. Taking the complex conjugate simply involves negating the imaginary component. For a real valued function this means no change. Correlation tells you how similar the signal is to the filter at any point.

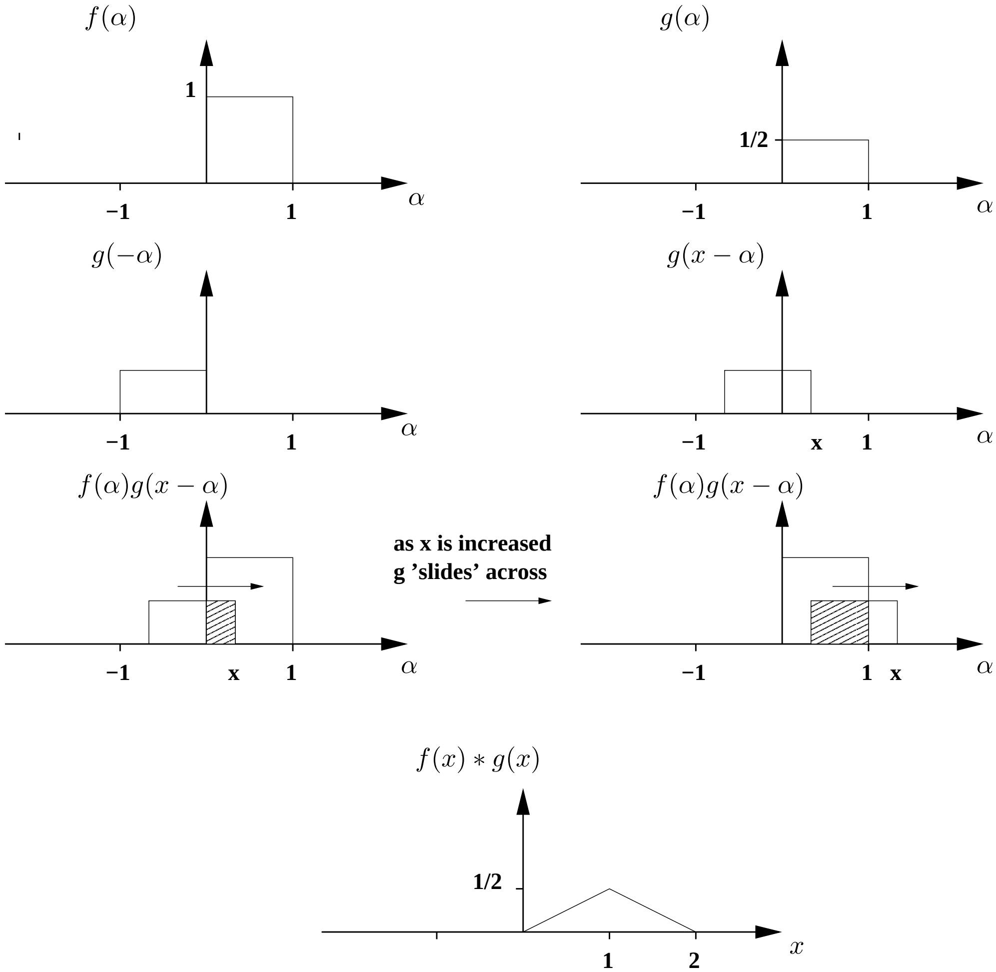

Figure 1: Graphical illustration of the convolution process. At the top we see the functions $f(\alpha)$ and $g(\alpha)$. The second row shows the flipped version of g, $g(-\alpha)$ and when g is shifted by x, $g(x-\alpha)$. The third row shows the inner product between $f(\alpha)$ and $g(x-\alpha)$, which lead to the final result shown at the bottom.

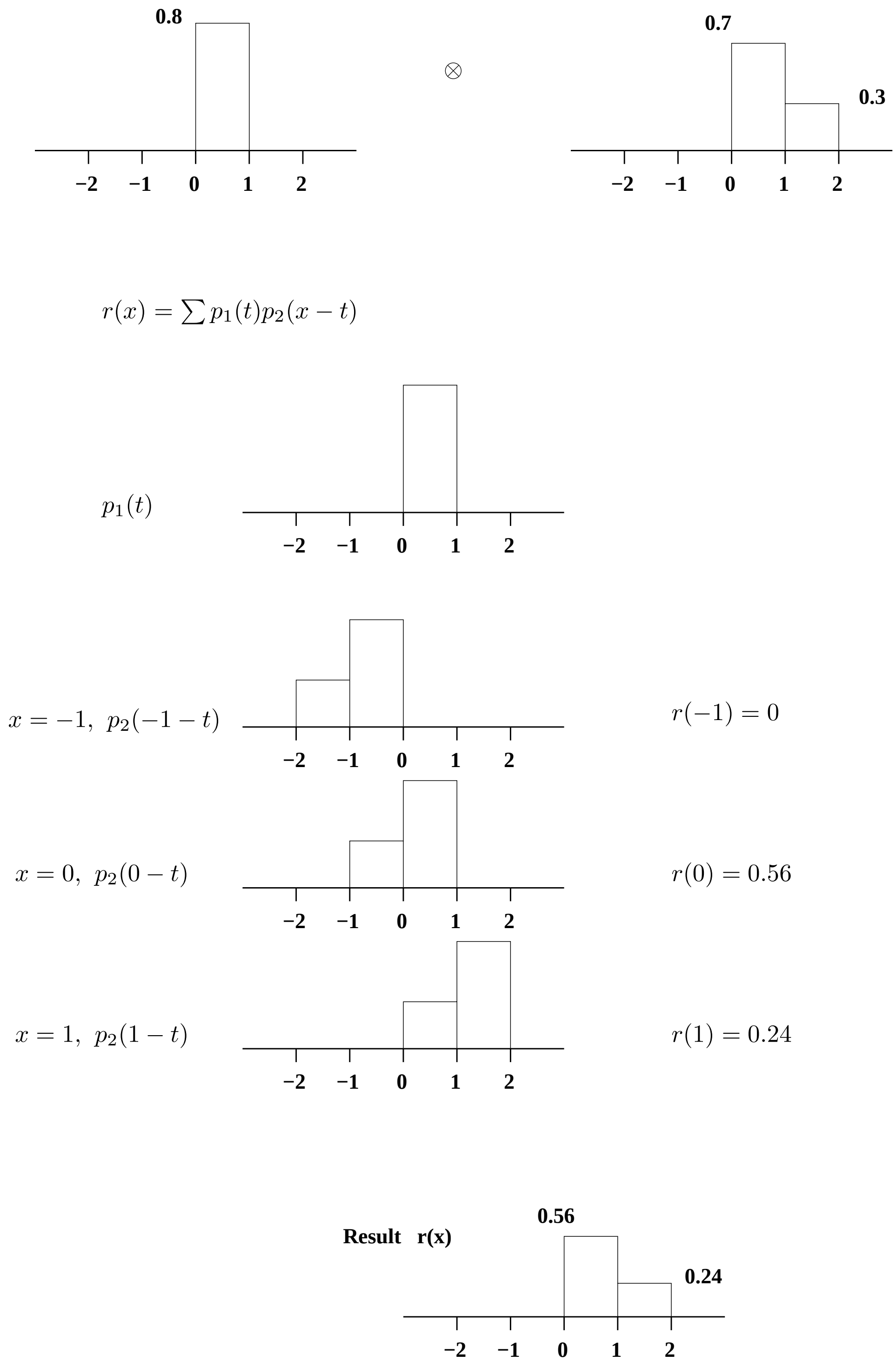

Figure 2: Graphical illustration of convolution in the discrete domain.

Correlation is similar to convolution except that one does not need to flip $g(x)$ about the origin. For real-valued functions where the second function (the filter) is symmetric convolution and correlation are identical. Where the second function is anti-symmetric the convolution will be the negative of the correlation. Almost all filtering functions used are either symmetric or anti-symmetric.

The term convolution is often used quite loosely in computer vision. Very often when people say convolution what they typically mean, or implement, is actually correlation. This usually does not matter because we usually work with real-valued data and with filters that are symmetric or anti-symmetric. However, be aware of this and look out for situations where the distinction is important, especially when using complex valued data.