Given that f is the signal (in the spatial domain, say) sampled at $x = 0, 1, \cdots, N-1$. The **Discrete Fourier Transform (DFT)** of f is computed in Matlab as follows:

$$F(\omega) = \sum_{x=0}^{N-1} f(x) e^{-i2\pi\omega x/N},$$

where ω is the frequency value. The inverse Discrete Fourier Transform (IDFT) is given by

$$f(x) = \frac{1}{N} \sum_{\omega=0}^{N-1} F(\omega) e^{i2\pi\omega x/N}.$$

Alternatively, since the frequency is periodic over the interval of the signal, we have

$$f(x) = \frac{1}{N} \sum_{\omega=-N/2}^{N/2-1} F(\omega) e^{i2\pi\omega x/N}. \qquad \text{if } N \text{ is even}$$

$$f(x) = \frac{1}{N} \sum_{\omega=-(N-1)/2}^{(N-1)/2} F(\omega) e^{i2\pi\omega x/N}. \qquad \text{otherwise.}$$

Note that there is a factor of $1/N$ for the IDFT. This factor is not present for the continuous case.

If, for a specific frequency value ω_0, we have $F(\omega_0) = a + bi$, where a and b are both scalars, then $F(-\omega_0) = a - bi$. If we zero out all the frequency components, leaving only $F(\omega_0)$ and $F(-\omega_0)$, and apply the IDFT, then we obtain the partial reconstruction

$$\begin{aligned}
\tilde{f}_{\omega_0}(x) &= \frac{1}{N} \sum_{\omega=0}^{N-1} F(\omega) e^{i2\pi\omega x/N} \\
&= \frac{1}{N} \left(F(\omega_0) e^{i2\pi\omega_0 x/N} + F(-\omega_0) e^{i2\pi(-\omega_0)x/N} \right) \\
&= \frac{1}{N} \Big((a + bi) \left(\cos(2\pi\omega_0 x/N) + i \sin(2\pi\omega_0 x/N)\right) + \\
&\qquad (a - bi) \left(\cos(2\pi\omega_0 x/N) - i \sin(2\pi\omega_0 x/N)\right) \Big) \\
&= \frac{1}{N} \Big[\big(a \cos(2\pi\omega_0 x/N) - b \sin(2\pi\omega_0 x/N)\big) + \big(a \sin(2\pi\omega_0 x/N) + b \cos(2\pi\omega_0 x/N)\big) i + \\
&\qquad \big(a \cos(2\pi\omega_0 x/N) - b \sin(2\pi\omega_0 x/N)\big) - \big(a \sin(2\pi\omega_0 x/N) + b \cos(2\pi\omega_0 x/N)\big) i \Big] \\
&= \frac{2}{N} \big(a \cos(2\pi\omega_0 x/N) - b \sin(2\pi\omega_0 x/N) \big).
\end{aligned}$$

Thus, using the frequency component at ω_0, the partial reconstruction, $\tilde{f}_{\omega_0}(x)$, is a linear combination of the two basis cosine and sine functions corresponding to ω_0. The complete reconstruction, $f(x)$, is the total of these partial reconstructions from all frequency components.